Week 8. Games with incomplete information

Bonus 1: Volunteer's dilemma with correlated costs

Consider the volunteer's dilemma with payoffs

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} (1-c_1, 1-c_2) & (1-c_1, 1) \\ (1, 1-c_2) & (0,0) \end{pmatrix} \end{array}$$

Suppose c_1 is randomly drawn from $[0,2]$ uniformly. For c_2 assume that it is (anti-) correlated, such that $c_1 + c_2 = 2$ is always satisfied (whenever player 1 has a comparably high cost they know player 2 has a comparable low cost). **Show that the following strategies are a Bayesian Nash equilibrium.**

$$s^{(1)}(c_1) = \begin{cases} C, & \text{if } c_1 \leq 1 \\ D, & \text{if } c_1 > 1 \end{cases} \qquad s^{(2)}(c_2) = \begin{cases} C, & \text{if } c_2 < 1 \\ D, & \text{if } c_2 \geq 1 \end{cases}$$

Bonus question: Are there any other Bayesian Nash equilibria?

[Hint: To show that $(s^{(1)}, s^{(2)})$ is a Bayesian Nash equilibrium, consider player 1. Show that for the three cases $c_1 < 1$, $c_1 > 1$ and $c_1 = 1$, the player has no reason to deviate from $s^{(1)}$. Do the same for player 2.]

Bonus 2: Second-price sealed bid auction

An item is sold to the highest bid among n players. The players' values for the item, $\mathbf{v}^{(i)}$, are uniformly and independently drawn from $[0,1]$. The players' strategy is to choose a bid, depending on how much the item would be worth to them, $s^{(i)} : \mathbf{v}^{(i)} \to b^{(i)}$.

The player with the highest bid wins the auction. However, they only have to pay the second-highest bid.

$$\text{Payoff: } \pi^{(i)}(b^{(i)}, b^{(-i)}) = \begin{cases} \mathbf{v}^{(i)} - \max\limits_{j \neq i}(b^{(j)}), & \text{if } b^{(i)} > b^{(j)} \ \forall\ j \neq i \\ (\mathbf{v}^{(i)} - \max\limits_{j \neq i}(b^{(j)}))/K, & \text{if } b^{(i)} = \max\limits_{j \neq i}(b^{(j)}) \ \&\ K\text{number of highest biddings} \\ 0, & \text{otherwise.} \end{cases}$$

Show that the strategy $b^{(i)} = v^{(i)}$ weakly dominates any other strategy.

[Hint: Consider 3 cases. The first case is that $\mathbf{v}^{(i)} > \max_{j \neq i}(b^{(j)})$. Show that bidding $b^{(i)} \neq \mathbf{v}^{(i)}$ never yields a higher payoff than bidding $b^{(i)} = \mathbf{v}^{(i)}$, but sometimes the payoff is worse. Similar for the cases $\mathbf{v}^{(i)} = \max_{j \neq i}(b^{(j)})$ and $\mathbf{v}^{(i)} < \max_{j \neq i}(b^{(j)})$.]

Exercise 1: Revenue equivalence theorem

Consider an auction between two players with valuations for an item again uniformly drawn from $[0, 1]$. We already know the equilibria of the

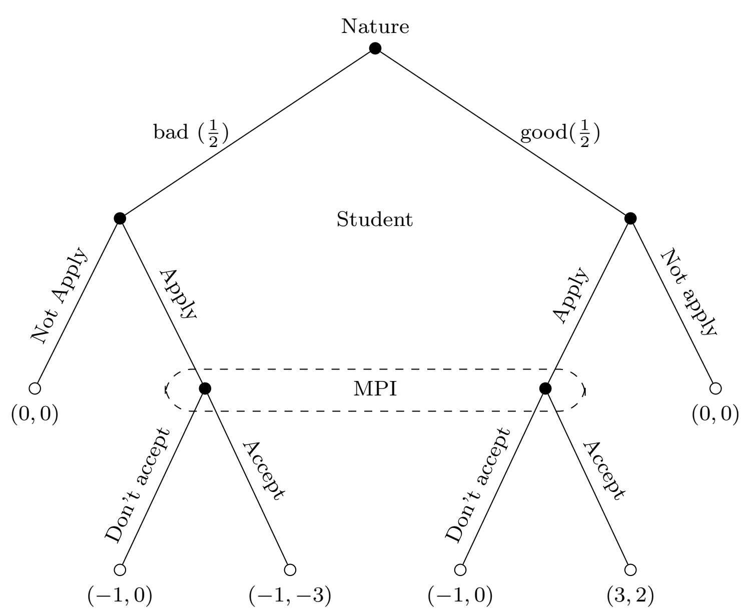
- first-price sealed bid auction: $b^{(i)} = \frac{1}{2}\mathbf{v}^{(i)}$ (Example 4.5), and the
- second-price sealed bid auction: $b^{(i)} = \mathbf{v}^{(i)}$ (Bonus 2).

Show that the expected revenue for the seller (the price the winning bidder has to pay) is the same for both auctions.

[Hint: Suppose without loss of generality that the player with the highest valuation assigns a value $\mathbf{v}$ to the item. Show that the expected revenue of the seller is $\frac{\mathbf{v}}{2}$, in both cases.]

Exercise 2: Pooling equilibrium of the IMPRS game

Revisit the IMPRS game from Example 4.8.



Show that if the players use the following strategies, players have no incentive to deviate.

$$\begin{aligned} s^{(1)}(\theta) &= \text{Not apply} \;\; \forall\, \theta \in \{\text{good, bad}\}, \\ s^{(2)} &= \text{Don't accept.} \end{aligned}$$

[Hint: For this, note that in this equilibrium, no student ever applies. So if MPI observes an application, it does not know from the student's strategies which of the student applied. Assume that MPI thinks $\mathbb{P}(\text{good} \mid \text{Apply}) = \frac{1}{2}$.]