GAME THEORY

§0 About this class

Remark 0.1: Motivation for this course

1) Give an introduction to classical game theory
   ⇝ Allow you to talk to economists, etc. and read their articles.

2) Brain food (not too many courses at MPI)

Remark 0.2: Prerequisites

1) Not too many: Calculus (Derivatives, integrals, infinite series), Some linear algebra

2) How to deal with math:
   (*) Try to give you the intuition for each concept
   (*) Try to show how this intuition can be translated into math. (Sometimes already intuitive concepts require quite some formalism to be made rigorous)
   ⇒ Not going to hide the math.

Remark 0.3: What to take out of this class

1) The concepts themselves.

2) How to translate intuitions into models

META 1

(Put yourself into the shoes of a smart math-person years ago)

(All of this is man-made ⇝ These are choices to be made)

3) How to teach this material

META 2

Remark 0.4 Some administrative remarks

Q&A

1) Lecture + Exercises: April 9, 16, 23, 30, May 7, 14, 21, 28

↳ 2x50 minutes

↳ 3-4 exercises per week

(*) Try to think first about each exercise on your own

(*) To solve all exercises, group work is fine (but let me know)

2) How to "pass" this course

(*) I hope you are here because you enjoy it, as it's useful

(*) For IMPRS: 3 day equivalents

(-) Solve $\gtrsim \frac{1}{2}$ of the exercises

(-) Explain some of the concepts to me.

---

§1 Introduction

What will we be talking about? Which topics are related but we will not talk about them?

Remark 1.1 (What is game theory)

Game theory is the field that explores

(Strategic) decision-making

Not GT: (*) How to play Roulette } "Decision
(*) Whether to buy a given flat } theory"

GT: (*) How to play poker
(*) How to bargain the price of a flat you want to buy

Remark 1.2 (Elements of game theory)

1) Who are the players?
2) What are the players' possible actions?
3) Which information do players have when making their moves?
4) What is the temporal order of moves?
5) What are the payoffs (i.e. what are the consequences of decisions)

Example 1.3 (Volunteer's dilemma: Cleaning the kitchen)

1) Players: Members of the Pink Residence $\mathcal{N} = \{1, \dots, n\}$
2) Possible actions: $A_i = \{\overset{C}{\text{Clean}}, \overset{D}{\text{Don't clean}}\}$ $\forall i \in \mathcal{N}$
$\mathcal{A} := A_1 \times A_2 \times \dots \times A_n$
3) Information: Players understand how much they appreciate a clean kitchen and how much other players appreciate a clean kitchen. They understand the effort it takes to clean.
They don't know which actions other players will choose.
Assumption: Same benefit b, same cost c.
4) Players decide simultaneously
5) Payoffs (map that assigns numerical outcomes to each possible combination of actions)

$$\Pi : \mathcal{A} \to \mathbb{R}^n \qquad (a_1, a_2, \dots, a_n) \longmapsto (\Pi_1, \Pi_2, \dots$$

$$\Pi_i(\vec{a}) = \begin{cases} b & \text{if } \exists j \neq i : a_j = C \text{ and } a_i = D \\ b - c & \text{if } a_i = C \\ 0 & \text{if } a_j = D \ \forall j \end{cases}$$

Example 1.4 (Poker)

1) Players: Whoever sits at the table $\mathcal{N} = \{1, \dots, n\}$
2) Actions: Simplified: $A_i = \{$ Fold, Check, Raise $\}$
3) Information:
   (*) Players know their own cards
   (*) know others' bidding behavior so far
   (*) They know how many cards there are,
   but they don't know which cards their co-players have
4) Players move alternatingly
5) Payoffs: Complicated.
   Depends on players' cards + Players' decisions
   Π : Cards $\times$ Decisions $\to \mathbb{R}^n$

Remark 1.5 (Distinction "action" vs. "strategy")

1) An action is what the player does
2) A strategy is a rule that tells the player what to do, given the information that she has.

<u>Question:</u> Now that we have defined these games, how do we solve them?
What does it even mean to "solve" a game?

Remark 1.6 (A map of game theories)

Game Theory

- **Experimental GT**: How do people play?
- **Classical game theory**
  - Idea: What would ration players do?
  - (*) Rational: Players understand th game, maximize their payoffs
  - (*) Common knowledge of rationa
- **Algorithmic game th.**
  - Idea: What is the computational complexity of solving games
- **Epistemic game theory**
  - Idea: Once there is no common knowledge of rationality how do players form expectations
- **Evolutionary game theory**
  - Idea: Players are not rational, but they try to improve their payoff over time

Remark 1.7 (Cooperative vs. non-cooperative GT)

(-) Cooperative game theory: Players can make costless binding agreements $\Rightarrow$ Outcome will always be efficient, but still question of how to allocate payoffs. $\rightsquigarrow$ coalitions etc

(-) Non-cooperative game theory: No binding agreements but individual strategies

Remark 1.8 (A short look on cooperative game theory)

(-) Setup: Players $\{1, \dots, n\}$

Value $v(i)$: payoff that i can guarantee herself

$v(C)$: payoff that a coalition C can guarantee itself

Allocation $x(i)$: Payoff that i gets in final allocation

(-) Next step is to define axioms on what is reasonable

(-) Example: Solution concept "Core"

A1: Individual rationality $x(i) \geq v(i)$

A2: Group rationality: For any $C \subseteq \{1,\dots,n\}$: $\sum_{i \in C} x_i \geq v(C)$

(-) Example: Glove game

3 players, first two have a left glove, third has a right glove

$$v(1) = v(2) = v(3) = 0, \quad v(\{1,2\}) = 0$$

$$v(\{1,3\}) = v(\{2,3\}) = v(\{1,2,3\}) = 1$$

Individual rationality $\rightarrow$ $x_1, x_2, x_3 \geq 0$

Group rationality $\rightarrow$ $x_1 + x_2 \geq 0$, $x_1 + x_3 \geq 1$, $x_2 + x_3 \geq 1$

$$x_1 + x_2 + x_3 = 1$$

Unique solution $(x_1, x_2, x_3) = (0, 0, 1)$

Could players 1 and 2 form a coalition to prevent this outcome? No, because suppose resulting allocation is $(1/4, 1/4, 1/2)$. Then 3 could offer 2 a share of $1/4 + \varepsilon$ ↯

Remark 1.9 (Outlook)

Next topic: normal form games [static games with complete information] and how to solve them.