GAME THEORY #3

A reminder

(*) Current topic: Static games with complete information (SGCI)

1 round, players move simultaneously

players have all relevant information

(*) Elements of SGCI:

Players: $\mathcal{N} = \{1, \dots, n\}$

Actions $\mathcal{A} = A^{(1)} \times A^{(2)} \times \dots \times A^{(n)}$

Payoffs: $\pi : \mathcal{A} \to \mathbb{R}^n$

(*) Strategies for SGCI

Probability distributions $\sigma^{(i)} = (\sigma_1^{(i)}, \dots, \sigma_k^{(i)})$

$\Sigma^{(i)}$ ... set of all strategies of player i

$\Sigma = \Sigma^{(1)} \times \Sigma^{(2)} \times \dots \times \Sigma^{(n)}$ strategy profiles

Extend payoff function such that $\pi : \Sigma \to \mathbb{R}^n$

Pure strategy $s_j^{(i)}$: Player i plays $a_j^{(i)}$ with probability

(*) Our task now: Given we have a game.

What would be a reasonable solution?

Abstractly: $\Psi : \Gamma = (\mathcal{N}, \mathcal{A}, \pi) \to \Sigma$

(*) Special case of prisoner's dilemma

	Silent	Confess
~~Silent~~	~~3,3~~	~~0,4~~
Confess	4,0	1,1

Example 2.13 (Prisoner's dilemma with remorse)

	Silent	Confess
~~Silent~~	~~3,3~~	~~0,0~~
Confess	4,0	1,1

Definition 2.14 (Iterated elimination of dominated strategies, IEDS)

Define recursively

$$S_0^{(i)} = S^{(i)}, \quad \Sigma_0^{(i)} = \Sigma^{(i)}$$

Define iteratively all strategies that are not dominated after k steps

$$S_k^{(i)} = \{ s^{(i)} \in S_{k-1}^{(i)} : \nexists \sigma^{(i)} \in \Sigma_{k-1}^{(i)} : \pi^{(i)}(\sigma^{(i)}, s^{(-i)}) > \pi^{(i)}(s^{(i)}, s^{(-i)}) \ \forall s^{(-i)} \in S_{k-1}^{(-i)} \}$$

$$\Sigma_k^{(i)} = \{ \sigma^{(i)} \in \Sigma^{(i)} : \sigma_j^{(i)} > 0 \Rightarrow s_j \in S_k^{(i)} \}$$

We call the game dominance solvable if for all players i the object $\bigcap_{k=0}^{\infty} S_k^{(i)}$ contains only one element.

Example 2.15 (Prisoner's dilemma with remorse)

Silent Confess

Silent	3,3	0,0
Confess	4,0	1,1

$$
\begin{array}{lll}
\underline{k=0} & \underbrace{S_0^{(1)} = \{\overset{(1,0)}{\text{Silent}}, \overset{(0,1)}{\text{Confess}}\}} & S_0^{(2)} = \{\text{Silent}, \text{Confess}\} \\
\underline{k=1} & S_1^{(1)} = \{\text{Confess}\} & S_1^{(2)} = \{\text{Silent}, \text{Confess}\} \\
\underline{k=2} & S_2^{(1)} = \{\text{Confess}\} & S_2^{(2)} = \{\text{Confess}\} \\
k>2 & \text{— " —} & \text{— " —}
\end{array}
$$

$$\bigcap_{k=0}^{\infty} S_k^{(1)} = \{\text{Confess}\} \qquad \bigcap_{k=0}^{\infty} S_k^{(2)} = \{\text{Confess}\}$$

Example 2.16 (Traveler's dilemma)

An airline needs to reimburse two travelers for having lost their (identical) souvenirs.

Each traveler needs to say a value $\{180, \dots, 300\}$

If travelers say different amounts, the one with the lower amount gets some reward $R = 5$.

$[180, 300] \in \mathbb{N}$

Game: $\mathcal{N} = \{\text{Traveler } 1, \text{Traveler } 2\}$

$$A^{(1)} = \{180, \dots, 300\}$$

$$\pi^{(1)}(a_1, a_2) = \begin{cases} a_1 & \text{if } a_1 = a_2 \\ a_1 + R & \text{if } a_1 < a_2 \\ a_2 & \text{if } a_1 > a_2 \end{cases}$$

$\underline{k=0}$ $\quad S_0^{(1)} = \{180, 181, \dots, 300\}$ $\quad S_0^{(2)} = \{180, \dots, 300\}$

$S_0 = \{180, \dots, 300\}$

<u>$k=1$</u> Claim: $a = 300$ is (weakly) dominated by 299

<u>Proof</u>: Case 1: Suppose co-player chooses $a_2 \leq 299$

$$\pi^{(1)}(300, a_2) = a_2 = \pi^{(1)}(299, a_2)$$

Case 2: Suppose co-player chooses $a_2 = 300$

$$\pi^{(1)}(300, 300) = 300$$
$$\wedge$$
$$\pi^{(1)}(299, 300) = 304$$

$S_1^{(1)} = \{180, \dots, 299\}$ $\qquad$ $S_1^{(2)} = \{180, \dots, 299\}$

<u>$k=2$</u> $\quad S_2^{(1)} = \{180, \dots, 298\}$ $\qquad$ $S_2^{(2)} = \{180, \dots, 298\}$

Arbitrary <u>$k \leq 120$</u> $\quad S_k^{(1)} \{180, \dots, 300-k\}$

$k > 120 \quad S_k^{(1)} = \{180\}$

IEDS is complex $\rightsquigarrow$ 2 more examples in exercises

---

<u>Remark 2.17</u> (On the status of IEDS within classical game theory)

(1) If a game is dominance solvable, the solution is usually considered as quite convincing.
Only requires Rationality + Common knowledge of rationality ✓

(2) Solutions are unique ✓

(3) Solutions may not exist ✗

Remark 2.18 (IEDS in other game theories)

(*) Epistemic game theory

R ... players are rational

KR ... players know they are rational

K^2R ... players know they know they are rational

To solve the traveler's dilemma you need $K^{99}R$

(*) Evolutionary game theory

Replicator dynamics: Infinite population, symmetric game with k pure strategies, x_j ... fraction of players using strategy s_j

$$\dot{x}_j = x_j \, [\underbrace{\pi_j}_{\text{Expected payoff of } s_j} - \underbrace{\bar{\pi}}_{\text{Average}}]$$

Theorem: Suppose the game is dominance solvable such that s_j is the solution. Then $x_j(t) \xrightarrow{t \to \infty} 1$ for all solutions of replicator dynamics with $x_j(0) > 0$

Hofbauer & Sandholm (2011) cool!

↳ Reading list

(*) Behavioral game theory:

Goeree & Holt (AER 2001)

Traveler's dilemma $\{180, 181, \dots, 300\}$ [cents]

$R = 180$ $\approx 80\%$ went for 180

$R = 5$ $\approx 80\%$ went for 300.

§ 2.2 Nash equilibria

$\Phi : \Gamma \to \Sigma$

Example 2.19 (Stag-hunt)

	Stag	Hare
Stag	10,10	0,6
Hare	6,0	6,6

This is not dominance solvable.

Somewhat unreasonable to call (Stag, Hare) a solution.

There is a sense in which (Stag, Stag) seems more reasonable.

Definition 2.20 (Nash equilibrium)

(1) Consider a game $\Gamma = (N, A, \pi)$. Then a strategy profile $\hat{\sigma}$ is a Nash equilibrium if for all players i
$= (\hat{\sigma}^{(1)}, \hat{\sigma}^{(2)}, \dots, \hat{\sigma}^{(n)})$

$\pi^{(i)}(\hat{\sigma}^{(i)}, \hat{\sigma}^{(-i)}) \geq \pi^{(i)}(\sigma^{(i)}, \hat{\sigma}^{(-i)})$

$\pi^{(i)}(\sigma^{(i)}, \sigma^{(-i)}) \leq \pi^{(i)}(\sigma^{(i)}, \sigma^{(-i)}) \quad \forall \sigma^{(i)}$

$\Sigma^{(1)} \quad \Sigma^{(2)}$

(2) The NE is called strict if the inequality is strict for all $\sigma^{(i)} \neq \hat{\sigma}^{(i)}$