GAME THEORY #5

Reminder

(*) Classical game theory is about strategic decision-making amongst rational actors

Elements: (1) Players (2) Actions (3) Order of moves (4) Information (5) Payoffs

(*) Static games with complete information: $\Gamma = (N, A, \pi)$

(*) Two solution concepts
(a) Dominance solvability
(b) Nash equilibrium [Given that others use s^{-i}, I want to use s^{i}]

(*) Example Stag Hunt

	Stag	Hare
Stag	10,10	0,6
Hare	6,0	6,6

2 Nash equilibria in pure strategies

§3 DYNAMIC GAMES WITH COMPLETE INFORMATION

§3.1 Games of perfect information (Sequential games)

Example 3.1 (Sequential stag hunt)

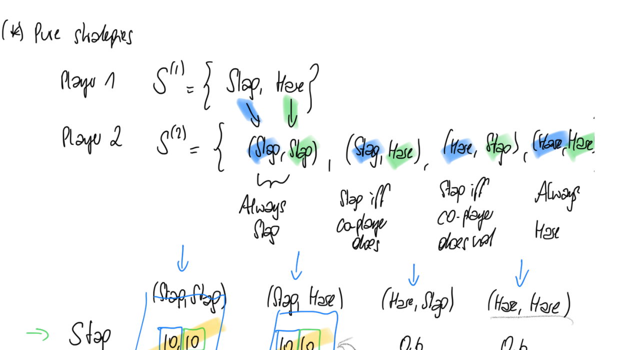
Players, actions, payoffs are as before. However, now player 1 makes her decision first and announces publicly what it is. Then player 2 decides.

→ Asymmetry: Player 2 knows player 1's move when making her decision.

(*) Pure strategies

Player 1: $S^{(1)} = \{Stag, Hare\}$

Player 2: $S^{(2)} = \{\ldots\}$: Always Stag, Stag iff co-player stag, Stag iff co-player hare, Always Hare



3 pure Nash equilibria

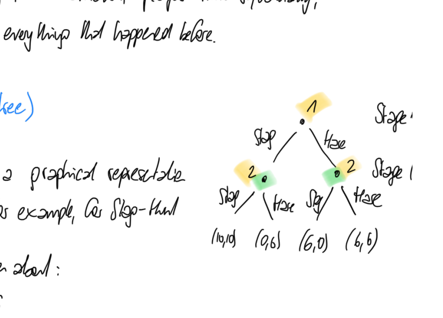
Question: Is this OK?

Definition 3.2 (Games of perfect information)

In a game of perfect information, players move sequentially, and they know everything that happened before.

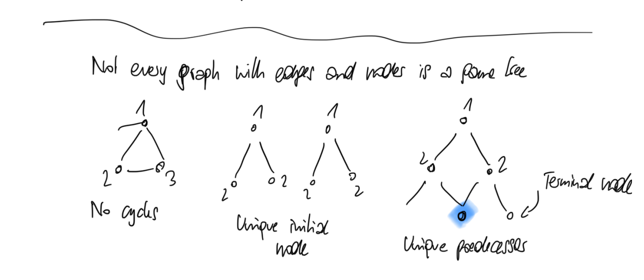
Remark 3.3 (Game trees)

A game tree is a graphical representation of the game. For example, for Stag-Hunt



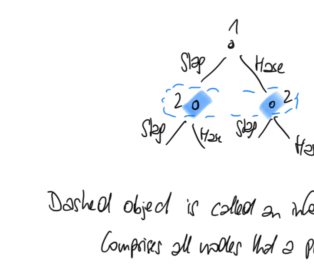
Contains information about:
(a) Who moves
(b) What is it that players know
(c) What are the potential outcomes

Not every graph with edges and nodes is a game tree



No cycles, Unique initial node, Unique predecessor

Side remark: One can also represent games with simultaneous moves. For example, classical Stag-hunt



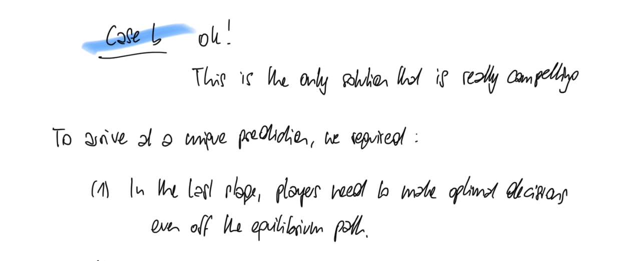
$S^{(1)} = \{Stag, Hare\}$, $S^{(2)} = \{Stag, Hare\}$

Dashed circle is called an information set. Comprises all nodes that a player cannot distinguish.

Overall, every game with complete information can be represented as a tree.

Example 3.4 (Stag-Hunt ctd.)

Let us illustrate the 3 Nash equilibria



Let's analyze the off-equilibrium behavior

Case a: Unreasonable to play Stag if I already know co-player chose Hare

Case c: Unreasonable to play Hare if I already know co-player chose Stag.

Case b: ok!
This is the only solution that is really compelling

To arrive at a unique prediction, we required:

(1) In the last stage, players need to make optimal decisions even off the equilibrium path.

(2) Players who make decisions at previous nodes can rely on (1)

Proposition 3.5 (Backward induction)

For any game with perfect information and finitely many stages one can find a Nash equilibrium with the following algorithm:
↳ Exercise

(1) For each node in the final stage, determine the optimal action of the respective player

(2) Go to the second-to-final stage. Determine the optimal behavior, given players in the final stage will behave optimally (see (1))

(3) Repeat until initial node.

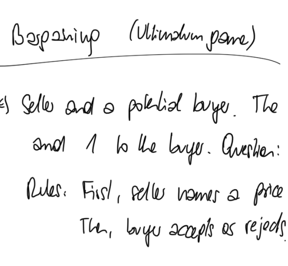
The solution has the property that players act optimally at every possible node.
For almost all games this solution is unique.

Examples 3.6

1) Cooperation and punishment

First, player 1 can decide whether or not to help player 2
(helping means: pay some cost $c>0$ to transfer a benefit $b>0$ to the co-player)

Second, player 2 can decide whether to punish non-helpers
(punishing means: pay some cost $\gamma>0$ to induce some damage $\beta>0$ on the co-player)



Backward induction:
(*) Player 2 would never punish
(*) Knowing this, player 1 would never help.

Punishment is a non-credible threat

2) Bargaining (Ultimatum game)

(*) Seller and a potential buyer. The item is worth 0 to the seller and 1 to the buyer. Question: Which price should they agree on?

Rules: First, seller names a price
Then, buyer accepts or rejects

(*) Players: $N = \{Seller, Buyer\}$

Actions: $A^{(1)} = \{0.01, 0.02, \ldots, 0.98, 0.99\}$ Possible price
$A^{(2)} = \{Accept, Reject\}$

Payoffs: $\pi(p, a^{(2)}) = \begin{cases} (p, 1-p) & \text{if } a^{(2)} = Accept \\ (0,0) & \end{cases}$

(*) Pure strategies

Seller $S^{(1)} = A^{(1)} = \{0.01, \ldots, 0.99\}$

Buyer $S^{(2)} = \{f : A^{(1)} \to \{Accept, Reject\}\}$

(*) There are many Nash equilibria

$s^{(1)} = 1/2$

$s^{(2)}(s^{(1)}) = \begin{cases} Accept & \text{if } s^{(1)} \le 1/2 \\ Reject & \text{otherwise} \end{cases}$

(*) What is the solution according to backward induction?

Buyer decides last

Seller proposes	Buyer
p = 0.01	Accept
p = 0.02	Accept
⋮	
p = 0.99	Accept

⇝ Optimal strategy for buyer: Always accept

Seller: Ask for the highest possible price, $p = 0.99$

⇝ Equilibrium: $s^{(1)} = 0.99$

$s^{(2)}(s^{(1)}) = Accept \quad \forall s^{(1)}$

"First-mover advantage"

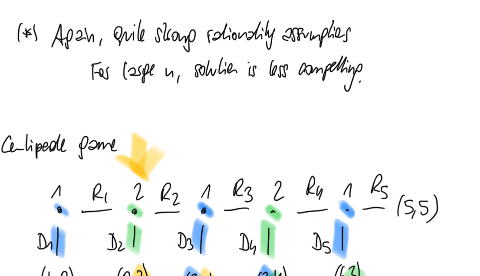
What happens if there is another round of negotiations?
⇝ Exercise

3) Stackelberg duopoly

Like Cournot, but now one firm moves first
⇝ Exercises

Remark 3.7 (Critique of backward induction)

1) Example:



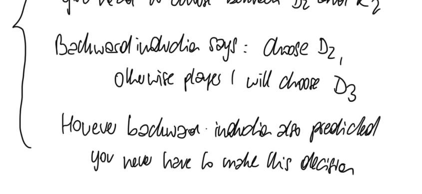
Backward induction: $\hat{\sigma} = (R_1, R_2, \ldots, R_n)$

(*) Suppose player 1 thinks there might be small probability ε to go down. To reach equilibrium, probability $(1-\varepsilon)^n$
⇝ Risky if there are many players.

(*) Player 2 might have similar considerations.
May reinforce player 1's temptation to exit

(*) Again, quite strong rationality assumptions
For large n, solution is less compelling.

2) Centipede game



Backward induction: At each node go down
Payoff (1,0) instead of (5,5)

(*) Do we find this solution compelling?

Backward induction paradox:

Suppose yes, and suppose you are player 2.
Suppose you find yourself in a situation where you need to choose between D_2 and R_2

Backward induction says: Choose D_2, otherwise player 1 will choose D_3

However backward induction also predicted you never have to make this decision

Player 2 might find it reasonable to play R_2
If player 1 anticipates this ⇝ might want to choose R_1 in the first place.

Remark 3.8 (Behavioral game theory and backward induction)

(*) Ultimatum game:

(-) Typically 40-50% of the surplus is offered to the buyer.

(-) If less than 20% of the surplus is offered ⇝ buyer rejects

(-) Huge cross-cultural variation

⇝ Richard McElreath

(*) Centipede game:

Usually players move quite far to the right
(⇝ ask Charlotte)

Palacios-Huerta & Volij: