Evolutionary Dynamics: Population Genetics and Ecological Models

WS 2019/2020 course number: MA4453-KP05

Assignment 1 due: October 28

Mendel's experiments & Punnett squares (4 P.)

In the lecture, we discussed Mendel's experiments with garden peas in which he considered one character (e.g. flower color or seed shape). In a second step, he considered two characters simultaneously. He started with two pure lines. One had yellow (dominant allele Y), wrinkled (recessive allele r) seeds. The other had green (recessive allele y), round (dominant allele R) seeds. I.e. the genotypes of these pure lines were $YYrr$ and $yyRR$. He crossed these two lines to obtain the F1 generation. He then obtained the F2 generation through selfing of individuals from the F1 generation. Draw the corresponding Punnett square to determine the genotypes and phenotypes that are generated in this way. What are the proportions of the observed phenotypes if you determine proportions by counting the respective cells in the Punnett square?

Mendel indeed observed these proportions. However, in 1905, William Bateson, Edith Rebecca Saunders, and Reginald Punnett performed a similar experiment in sweet pea plants. They simultaneously considered flower color and pollen shape and found different proportions than Mendel. How can this discrepancy be explained?

You can read Mendel's original publication here (in German!):
http://www.deutschestextarchiv.de/book/view/mendel_pflanzenhybriden_1866?p=14

Assortative mating (4 P.)

In the lecture, we studied randomly mating individuals. However, in reality, mating is often non-random. E.g. individuals may prefer to mate with individuals that are similar to them with respect to some trait. Let us look at a trait that is controlled by a single gene. For this gene, two different alleles, A and a, exist in a population of diploid individuals. A is dominant, i.e. individuals with genotype Aa have the same phenotype as individuals with genotype AA. A proportion ρ of individuals mate assortatively, i.e. with individuals with the same phenotype. Denote the genotype frequencies by P_{AA}, P_{Aa}, and P_{aa} and the allele frequencies by p and q.

1. Show that (in the absence of selection) the frequency of genotype AA in the next generation is given by
$$P'_{AA} = (1-\rho)p^2 + \rho\frac{p^2}{1-P_{aa}}.$$
Hint: Consider separately the contributions of those individuals that mate randomly and those that mate assortatively.

2. Derive the recursion equations for the frequencies of the other two genotypes.

3. How do the allele frequencies change from generation to the next?

Selection in diploids (4 P.)

Consider the special case that the fitness effects of alleles multiply to determine the fitness of a diploid individual, i.e. $w_{AA} = v_A v_A$, $w_{Aa} = v_A v_a$, $w_{aa} = v_a v_a$. Given the allele frequency $p_A(t)$ of allele A in generation t, determine the allele frequency in generation $t+1$ (assuming random mating). Compare to the haploid case. (Haploid individuals would have fitnesses v_A and v_a.) What do you observe? Is selection more effective in haploid or in diploid organisms, or is it equally effective?