Evolutionary Dynamics: Population Genetics and Ecological Models

WS 2019/2020 course number: MA4453-KP05

Assignment 2 due: November 11

Transition probabilities (4 P.)

Consider a population of size N that reproduces according to the Wright-Fisher model. There are two neutral alleles, A and a, segregating in the population.

- Show that the transition probabilities from state i to state j conditioned on fixation of allele A are given by

$$p^{\star}_{ij} = \binom{N-1}{j-1} \left(\frac{i}{N}\right)^{j-1} \left(\frac{N-i}{N}\right)^{N-j}.$$

- Calculate the expected change in the freqency of allele A from one generation to the next.

Single-generation identity by descent with separate sexes (4 P.)

Consider a population of diploid individuals consisting of N_f breeding females and N_m breeding males. Pick now two copies of an autosomal gene at random from two individuals in the offspring generation. What is the probability that they are descendants from the same ancestor in the previous generation? Now assume (1) that there is only one breeding male but all females in the population breed or (2) that all males and all females breed. What to you get for these special cases? Assume for these special cases that the sex ratio is 1:1.

Time to fixation in the neutral Moran model (4 P.)

Consider a neutral Moran model with two alleles (A and a) and discrete time steps. The total population size is N. Let τ_i be the expected time until one of the alleles fixes in the population given the number of A-alleles is i at time 0. Show that

$$\tau_{i-1} - 2\tau_i + \tau_{i+1} = -\frac{N^2}{i(N-i)}.$$

What are the boundary conditions?

The equation can be solved to give

$$\tau_i = N\left(\sum_{a=1}^{i} \frac{N-i}{N-a} + \sum_{a=i+1}^{N-1} \frac{i}{a}\right).$$

In the limit of large N, this can be approximated by

$$\tau(p) \approx -N^2 \left(p\ln(p) + (1-p)\ln(1-p)\right)$$

with $p = i/N$.

Compare to the result for the Wright-Fisher model derived in the lecture. What do you observe?