Evolutionary Dynamics: Population Genetics and Ecological Models

WS 2019/2020 course number: MA4453-KP05

Assignment 3 due: November 25

Probability generating functions (4 P.)

Let the initial spread of a benefical allele A be described by a Galton-Watson branching process. The offspring distribution of an individual carrying allele A is defined by the following probability generating function:

$$f(x) = \frac{1}{4} + \frac{1}{4}\,x + \frac{1}{2}\,x^2.$$

1. Assume there is exactly one individual carrying allele A in generation 0. What is the probability that the allele goes extinct at the first generation?

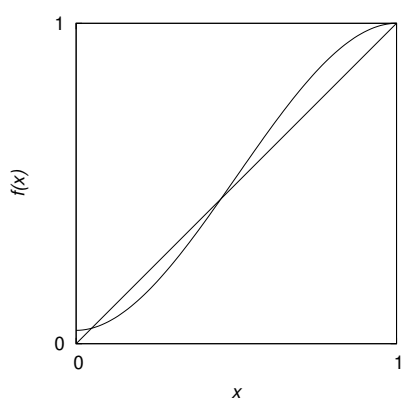
2. Write down the probability generating function of the number of A alleles in generation 2. (Algebraic simplification not necessary.)

3. Calculate the mean number of offspring for this distribution.

4. Assume there is initially one carrier of allele A. What is the probability that the allele gets established in the population?

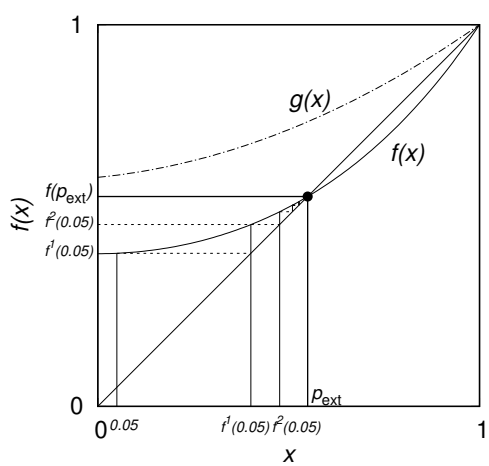
The shape of the probability generation function (4 P.)

In the lecture, we showed that the extinction probability is smaller than 1 when the mean of the offspring distribution is larger than 1, and it is 1 when the mean is ≤ 1. But why can some arbitrary probability generating function not have the following form?



Here the slope of f at $x = 1$ is less than one, but there are two solutions of the fixpoint equation $q = f(q)$ with $q < 1$.

Continuous-time birth-death processes (4 P.)

Consider a continuous-time birth-death process in which individuals reproduce at per-capita rate λ and die at per-capita rate μ.

- What is the probability distribution for the number of offspring that an individual produces over its lifetime? What is the expected number of offspring of an individual?
- Now consider a branching process in discrete time with this offspring distribution. Derive the corresponding probability generating function $f(y)$. From this, determine the establishment probability, assuming that there is exactly one individual in generation 0.