Evolutionary Dynamics: Population Genetics and Ecological Models

WS 2019/2020 course number: MA4453-KP05

Assignment 5 due: January 20

Changes in birth and death rates and evolutionary rescue (4 P.)

Imagine a population of haploid individuals where the wildtype has birth rate $\lambda_a < \mu_a$ such that the population size declines. A rescue mutant with $\lambda_A > \mu_A$ can have an increased birth rate or a reduced death rate (or both might be different from the wildtype traits). Explore how this influences the probability of evolutionary rescue by *de novo* mutations. Is the probability of rescue higher if the mutation increases the birth rate or if it decreases the death rate or is it equal in both cases?

Rescue from pre-existing mutants (4 P.)

A population gets exposed to a very harsh change in its environment that immediately kills all wild-type individuals. Mutants that can rescue the population may pre-exist in the population. In the new environment, mutant individuals have a Poisson distributed number of offspring with mean $1+s$. Denote by $\{p_k\}$, $k = 0, 1, 2, \dots$ the probability distribution for the number of mutants prior to the environmental change, i.e. p_k is the probability that there are k mutants in the population when the environment changes.

- Give the formula for the probability of evolutionary rescue under these circumstances.
- Now set $p_0 = 0.2$, $p_1 = 0.3$, $p_{100} = 0.5$, and $s = 0.01$. What is the probablity of evolutionary rescue?

The evolution of antibiotic resistance (4 P.)

Consider a bacterial population of N_0 sensitive bacteria that is treated with antibiotics. The antibiotic concentration is denoted by c. The per-cell death rate is μ. The per-cell birth rate is $\lambda_s(c) = \mu + S_s(c)$ with

$$S_s(c) = S^s_{\max} - \frac{(S^s_{\max} - S_{\min})\left(\frac{c}{c^s_{\text{MIC}}}\right)^\kappa}{\left(\frac{c}{c^s_{\text{MIC}}}\right)^\kappa - \frac{S^s_{\max}}{S_{\min}}}.$$

(It holds that $S_{\max} > 0$ and $S_{\min} < 0$.)

- What is the minimal concentration required to cause a population decline?

Through mutation, resistant bacteria may appear. The mutation probability at cell division is u. Assume that resistant bacteria have the same death rate as sensitive bacteria. But their birth rate is different and given by $\lambda_r(c) = \mu + S_r(c)$ with

$$S_r(c) = S^r_{\max} - \frac{(S^r_{\max} - S_{\min})\left(\frac{c}{c^r_{\text{MIC}}}\right)^\kappa}{\left(\frac{c}{c^r_{\text{MIC}}}\right)^\kappa - \frac{S^r_{\max}}{S_{\min}}}.$$

- Determine the probability of resistance evolution (= evolutionary rescue).

Now use the following parameter values: $S^s_{\text{max}} = 0.5$, $S^r_{\text{max}} = 0.49$, $S_{\text{min}} = -0.2$, $\kappa = 2$, $c^s_{\text{MIC}} = 5$, $c^r_{\text{MIC}} = 20$, $\mu = 1.0$, $N_0 = 10^9$, $u = 10^{-9}$.

- What is the probability of resistance evolution for $c = 5.5$, $c = 10$, $c = 25$?

In this scenario, the probability of resistance evolution decreases with the antibiotic dose c.

- Give one reason why this might not always be true.