Evolutionary Dynamics: Population Genetics and Ecological Models

WS 2019/2020 course number: MA4453-KP05

Assignment 6 due: February 3

Local stability analysis of models in continuous time (4 P.)

Consider a system in continuous time that is described by

$$\frac{\mathrm{d}n}{\mathrm{d}t} = f(n).$$

Let $\hat{n}$ be an equilibrium.

- Show that the dynamics of the displacement $\epsilon(t) = n(t) - \hat{n}$ can for small ϵ be approximated by
$$\epsilon(t) = e^{\rho t}\epsilon(0)$$
with $\rho = \left.\frac{\mathrm{d}f}{\mathrm{d}n}\right|_{n=\hat{n}}$.
- What can you conclude about the stability of the equilibrium depending on ρ?

Now consider specifically the continuous-time logistic model

$$\frac{\mathrm{d}n}{\mathrm{d}t} = r_c \cdot \left(1 - \frac{n(t)}{K}\right) \cdot n(t).$$

The model has two equilibria, $\hat{n}_1 = 0$ and $\hat{n}_2 = K$.

- Are these equilibria stable or unstable? How does this depend on r?
- Consider the equilibrium $\hat{n}_2 = K$ and $r < 0$. How does the system behave if the initial population size is larger than K? Does this make sense?

Exponential growth with declining growth rate (4 P.)

Consider a population whose growth rate is linearly declining due to environmental change. The population dynamics are described by

$$\frac{\mathrm{d}n}{\mathrm{d}t} = (r_0 - \delta t)\, n.$$

Use the method of separation of variables to solve this differential equation. How long does it take until the population has gone extinct (i.e. drops below 1 individual) if the initial population size is n_0?

Predator-prey model with two prey species (4 P.)

Set up a model for a predator and two prey species, where the two prey species compete for resources. Explain your choices. Write down the differential equations that describe how the respective population sizes change over time. (You do not need to solve them.)