Evolutionary Dynamics: Population Genetics and Ecological Models

WS 2019/2020 course number: MA4453-KP05

Assignment 7 due: February 11

The endemic equilibrium in the SIRS model (4 P.)

We introduced the SIRS model in the lecture:

$$\begin{aligned}\frac{\mathrm{d}S}{\mathrm{d}t} &= \Lambda - \delta_S S - \beta SI + qR\\ \frac{\mathrm{d}I}{\mathrm{d}t} &= \beta SI - \delta_I I - rI\\ \frac{\mathrm{d}R}{\mathrm{d}t} &= rI - \delta_R R - qR.\end{aligned}$$

- Calculate the endemic equilibrium.
- Compare the number of susceptibles at teh endemic equilibrium to their number in the SIR model. What do you observe? Give an intuitive explanation.

Total number of infected individuals (4 P.)

Consider the following special case of an SIR model (no births & no deaths):

$$\begin{aligned}\frac{\mathrm{d}S}{\mathrm{d}t} &= -\beta SI\\ \frac{\mathrm{d}I}{\mathrm{d}t} &= \beta SI - rI\\ \frac{\mathrm{d}R}{\mathrm{d}t} &= rI.\end{aligned}$$

- Sketch the dynamics of the number of susceptible, infected, and recovered individuals over time, starting with one infected individual.
- Denote by I_{total} the total number of individuals who had contracted the disease at the end of the epidemic (i.e. once it is over). How can you determine this number from the graph drawn before?
- Determine I_{total} mathematically. For this, look at the ratio $\frac{\mathrm{d}S/\mathrm{d}t}{\mathrm{d}R/\mathrm{d}t}$. (You will end up with a transcendental equation for I_{total} that can only be solved numerically. You do not have to do this.)

Vaccination (4 extra P.)

Consider a disease with $R_0 = 4$. A vaccine to this disease exists but its effectivity is only 90%, i.e. one out of ten vaccinated individuals is not immune despite being vaccinated. What is the minimum fraction of a population that needs to be vaccinated to prevent spread of the disease?